Projekt „Uruchomienie unikatowego kierunku studiów Informatyka Stosowana odpowiedzią na zapotrzebowanie rynku pracy"
jest współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego.

Numerical methods

Handouts for students

6. Polynomial interpolation

6.1. Lagrange interpolation polynomial

6.2. Newton interpolation polynomial

przygotowane w ramach projektu „Uruchomienie
go kierunku studiów Informatyka Stosowana odpowiedzią na
wanie rynku pracy" ze środków Programu Operacyjnego
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I. Introductory requirements

It is required to know the concepts of::

- polynomials;

and be able to:

- conduct operations on polynomials.

II. Classes

Task 1. Give three different examples of polynomials satisfying conditions: $W(0) = 0, W(-1) = -1, W(1) = 1$.

Task 2. Using Lagrange and Newton interpolating polynomials find polynomials of the smallest degree satisfying the following conditions (note that it is timesaving to use previous results to obtain next polynomials). Compare the amount of time required by both methods:

a) $\mathrm{W}(-1) = -4, \mathrm{W}(0) = -1, \mathrm{W}(1) = 6$;
b) $\mathrm{W}(-1) = -4, \mathrm{W}(0) = -1, \mathrm{W}(1) = 6\ \mathrm{W}(2) = 41$;
c) $\mathrm{W}(-2) = -75, \mathrm{W}(-1) = -4, \mathrm{W}(0) = -1, \mathrm{W}(1) = 6, \mathrm{W}(2) = 41$;
d) $\mathrm{W}(-2) = -75, \mathrm{W}(-1) = -4, \mathrm{W}(0) = -1,\ \mathrm{W}(1) = 6,\ \mathrm{W}(2) = 41\ , \mathrm{W}(3) = 200$.

Task 3. By means of Lagrange and Newton interpolating polynomials find polynomials of the smallest degree satisfying the following conditions:

a) $\mathrm{W}(-2) = 25, \mathrm{W}(1) = 19, \mathrm{W}(3) = 55$;
b) $\mathrm{W}(-1) = 1, \mathrm{W}(1) = 3, \mathrm{W}(5) = 127, \mathrm{W}(6) = 253$;
c) $\mathrm{W}(-2) = 16, \mathrm{W}(-1) = -4, \mathrm{W}(0) = -2, \mathrm{W}(1) = -8, \mathrm{W}(2) = -28$.

Task 4. You have a map that shows the location of a lost pirate treasure. According to the map you are currently at the point of coordinates $(0,1)$, and the treasure is hidden at the point $(4,41)$. In addition, it is noted on the other side of the map that the path to the treasure is full of traps and the only safe passage is through the points of coordinates $(1,-1), (2,-7), (3,-5)$. Use the formulas for the Lagrange and Newton interpolation to find the safe path from the start point to the place where the treasure is hidden.

Task 5. On days $1,2,3,5,6$ a weather station located in the village of Great Numerica measured the following amounts of rainfall $54,40,12,10,24\ mm/m^2$. On day 4 the measure was not performed due to the National Day. Use the formulas for Lagrange and Newton interpolation and verify whether the people of Great Numerica experienced rain on the National Day.

III. Homework

Task 1. Using Lagrange and Newton interpolating polynomials find polynomials of the smallest degree satisfying the following conditions:

a) $W(-3) = -5, W(0) = 1, W(1) = -1, W(3) = 25$;
b) $W(-1) = -1, W(3) = -5, W(4) = -16, W(6) = -92$;
c) $W(-2) = -21, W(-1) = -1, W(0) = 1, W(1) = 3, W(2) = -1$;
d) $W(-3) = 210, W(-1) = -8, W(1) = -26, W(3) = -324, W(5) = -614$.

Task 2. The *Metodus* probe, located in the orbit of Mars, searches the surface of the planet, looking for potential routes for the Mars rover *Numericus*. During the last contact with the Earth probe sent the following set of data: $(-3,-712), (-2,-104), (0,2), (1,-8), (2,-12), (3,176)$, where the first coordinate corresponds to a point on the investigated route, while the second – the height above a reference point. Use the formulas for Lagrange and Newton interpolation to calculate the height in the coordinate -1, where the *Numericus* currently is.

IV. Answers

Task 1.

a) Lagrange form: $-5\frac{(x-0)(x-1)(x-3)}{(-3-0)(-3-1)(-3-3)} + \frac{(x+3)(x-1)(x-3)}{(0+3)(0-1)(0-3)} - \frac{(x+3)(x-0)(x-3)}{(1+3)(1-0)(1-3)} + 25\frac{(x+3)(x-0)(x-1)}{(3+3)(3-0)(3-1)}$;
Newton form: $-5 + 2(x+3) - (x+3)x + (x+3)x(x-1)$;
Canonical form: $x^3 + x^2 - 4x + 1$;

b) Lagrange form: $-\frac{(x-3)(x-4)(x-6)}{(-1-3)(-1-4)(-1-6)} - 5\frac{(x+1)(x-4)(x-6)}{(3+1)(3-4)(3-6)} - 16\frac{(x+1)(x-3)(x-6)}{(4+1)(4-3)(4-6)} - 92\frac{(x+1)(x-3)(x-4)}{(6+1)(6-3)(6-4)}$;
Newton form: $-1 - (x+1) - 2(x+1)(x-3) - (x+1)(x-3)(x-4)$;
Canonical form: $-x^3 + 4x^2 - 2x - 8$;

c) Lagrange form: $-21\frac{(x+1)(x-0)(x-1)(x-2)}{(-2+1)(-2-0)(-2-1)(-2-2)} - \frac{(x+2)(x-0)(x-1)(x-2)}{(-1+2)(-1-0)(-1-1)(-1-2)} + \frac{(x+2)(x+1)(x-1)(x-2)}{(0+2)(0+1)(0-1)(0-2)} + 3\frac{(x+2)(x+1)(x-0)(x-2)}{(1+2)(1+1)(1-0)(1-2)} - \frac{(x+2)(x+1)(x-0)(x-1)}{(2+2)(2+1)(2-0)(2-1)}$;
Newton form: $-21 + 20(x+2) - 9(x+2)(x+1) + 3(x+2)(x+1)x - (x+2)(x+1)x(x-1)$;
Canonical form: $-x^4 + x^3 + x^2 + x + 1$;

d) Lagrange form: $210\frac{(x+1)(x-1)(x-3)(x-5)}{(-3+1)(-3-1)(-3-3)(-3-5)} - 8\frac{(x+3)(x-1)(x-3)(x-5)}{(-1+3)(-1-1)(-1-3)(-1-5)} - 26\frac{(x+3)(x+1)(x-3)(x-5)}{(1+3)(1+1)(1-3)(1-5)} - 324\frac{(x+3)(x+1)(x-1)(x-5)}{(3+3)(3+1)(3-1)(3-5)} - 614\frac{(x+3)(x+1)(x-1)(x-3)}{(5+3)(5+1)(5-1)(5-3)}$;
Newton form: $210 - 109(x+3) + 25(x+3)(x+1) - 10(x+3)(x+$
1x−1+2(x+3)(x+1)(x−1)(x−3);
Canonical form: $2x^4 - 10x^3 - 25x^2 + x + 6$.

Task 2. Lagrange form:
$-712\frac{(x+2)(x-0)(x-1)(x-2)(x-3)}{(-3+2)(-3-0)(-3-1)(-3-2)(-3-3)} - 104\frac{(x+3)(x-0)(x-1)(x-2)(x-3)}{(-2+3)(-2-0)(-2-1)(-2-2)(-2-3)} + 2\frac{(x+3)(x+2)(x-1)(x-2)(x-3)}{(0+3)(0+2)(0-1)(0-2)(0-3)} - 8\frac{(x+3)(x+2)(x-0)(x-2)(x-3)}{(1+3)(1+2)(1-0)(1-2)(1-3)} - 12\frac{(x+3)(x+2)(x-0)(x-1)(x-3)}{(2+3)(2+2)(2-0)(2-1)(2-3)} + 176\frac{(x+3)(x+2)(x-0)(x-1)(x-2)}{(3+3)(3+2)(3-0)(3-1)(3-2)}$;
Newton form: $-712 + 608(x+3) - 185(x+3)(x+2) + 41(x+3)(x+2)x - 7(x+3)(x+2)x(x-1) + 2(x+3)(x+2)x(x-1)(x-2)$;
Canonical form: $2x^5 - 3x^4 - x^3 - 3x^2 - 5x + 2$.

$$f(-1) = 0.$$